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The Strength of Polymeric Composites Containing Spherical Fillers

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Synopsis

A theoretical relationship has been developed which relates the ultimate strength of a composite containing spherical fillers to the size, volume fraction, and surface adhesion of the dispersed phase. The theoretical predictions are compared to experimental data using glass beads of known diameters in polyester resin matrix. Results were compared for the case of poor adhesion between the glass beads and the matrix and for the case of good adhesion. The derived relationships should be useful in helping to optimize the strength properties of particulate reinforced systems.

INTRODUCTION

Although there are a number of theories describing the elastic modulus behavior of filled polymer systems, a satisfactory treatment of the strength behavior of composites reinforced with rigid fillers has not yet been developed.

Nielsen,[1] in his review of the mechanical properties of particulate-filled systems, wrote: "Except for the case of filled rubbers, practically there is no good theory to guide one's thinking on the stress-strain properties of such materials. Empirically, it is known what will often happen, but the reasons for the observed behavior are often not clear." Nielsen,[1,2] using very simple models, gave semiquantitative prediction of the ultimate strength of composites in relation to the filler concentration for the case of perfect adhesion, and also no adhesion, between the filler and polymer phases. Nielsen's equations, however, do not include the effect of filler diameter on the ultimate strength of the composite.

Nicolais and Narkis[3] obtained an equation for the yield strength of polymer-filler systems which is very similar to that proposed by Nielsen.

Sahu and Broutman[4] used finite element analysis in order to predict the mechanical properties of thermosetting resins filled with glass microspheres. Although this method gives results in relatively good agreement with experiments, it is usually desirable to have not only a numerical solution but also an analytical one related to the composite parameters.

There have been a number of attempts to correlate the strength of a particulate-filled systems at a constant volume fraction of filler with the diameter of the filler particles as, for example, in the published work of Hajo and Toyoshima.[6] It has been shown in these publications that the ultimate strength of the composite was a linear function of the reciprocal of the square root of the diameter of the filler particles.

Alter[7] also related the strength of the composite to the reciprocal of the diameter of the filler.

The following treatment expresses the ultimate strength of a polymer filled with rigid spheres as a function of the volume fraction, the diameter, and the interfacial adhesion.

EXPERIMENTAL

Polyester resin (Stypol 40-2364, Freeman Chemical Co.) was used in all the examples. The resin was cured with 1% of a 60% solution of methyl ethyl ketone peroxide in dimethyl phthalate (Lupersol DDM, Lucidol Chemicals) catalyzed with 0.3% of a 6% solution of cobalt naphthenate in mineral spirits. Properties of the cured polyester resin are: tensile strength, 8,300 psi; flexural strength, 19,500 psi; shear strength, 4,150 psi (assumed to be half of the tensile strength); specific gravity, 1.13. Table I specifies glass beads (Potters Industries Inc.) used as fillers. The specific gravity of the beads was 2.48. Surface finish of the spheres was: no coating, or CPOl coating—proprietary coating recommended for use with polyester resins (Potters Industries Inc.)

The composites were prepared by mixing the resin with the curing agents and the glass beads, degassifying under vacuum for 5 min, and then casting the mixture between two glass plates separated by strips of rubber. In order to prevent settling of the beads, the mold was inverted at frequent intervals. The resin was cured for 20 min at 85°C and then postcured at 100°C for 4 hr.

The nominal volume percentage of glass spheres was 10–50%; the actual weight percentage was determined, after the sample had been cured, by ashing techniques from which the volume fractions could be calculated from the known specific gravity of the spheres and the resin.

The tensile and flexural strengths of the composites were measured on an Instron testing machine at a cross-head speed of 0.5 cm/min.

For the tensile strength, two specimens were tested to obtain the average value, and for the flexural strength, three specimens were tested. Me-

TABLE I
Glass Beads Used as Fillers

Stock size	U.S. Sieve no.	Diameter, mm
1721	45–80	0.354–0.177
1922	60–100	0.250–0.149
2024	70–140	0.210–0.105
2429	140–270	0.105–0.053
2900	minus 270	–0.053
3000	minus 325	–0.044

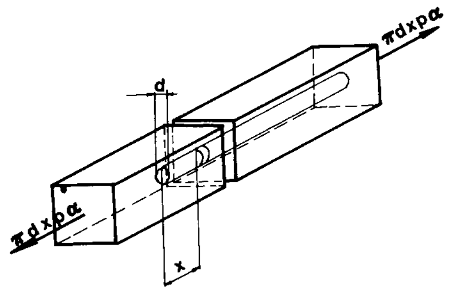


Fig. 1. Specimen prepared for the determination of frictional stress at matrix–filler interface. Force required to pull out the glass rod is equal to $\pi dxp\alpha$.

chanical properties of the unfilled resin test specimens were based on five separate measurements.

In order to obtain the value of the frictional stresses at the filler–matrix interface τ_x, a test specimen as illustrated in Figure 1 was prepared. Using an Instron tester, a glass rod was pulled out of the polyester resin in which it was embedded. From the measured load–displacement curve, frictional stresses at the glass rod–resin interface were calculated to be 1540 psi (in case of no adhesion).

THEORETICAL

In order to derive an analytical expression for the ultimate strength of the composite filled with spheres, the following approach was used. The well-established theory of Kelly[8] and Outwater[9] for fiber reinforcement was applied to the case of spheres after certain modifications. This way, the stress distribution in the beads, at the breaking point of the composite, was obtained. Average stress in the beads combined with volume fraction of the beads expressed the maximum load carried by the filler.

In order to calculate the maximum load carried by the matrix, it was assumed that the ultimate strength of the matrix itself is affected by the presence of the filler. This assumption is justified since the solid inclusions act as stress concentrators (Griffith's cracks).

The ultimate strength of the composite is simply the sum of the maximum load carried by both the matrix and the filler.

No Adhesion

Figure 2 shows the model which was used in the calculation of the stress distribution in the beads. A sphere has been approximated by a series of cylinders which makes possible the application of the well-known fiber reinforcement theories.[8,9]

Since there is no adhesion in this case, the stress is transferred from the matrix to the sphere by frictional forces such that

$$\tau_x = p_y \alpha \tag{1}$$

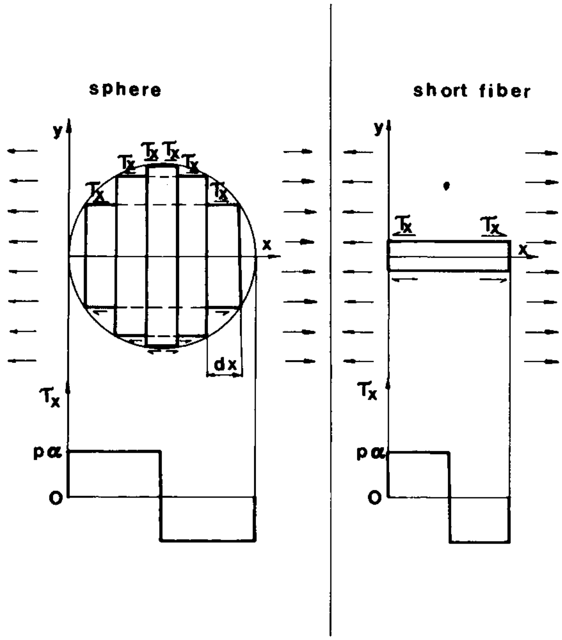


Fig. 2. Diagram of model used in developing the relationship for stress distribution in a spherical bead utilizing the short fiber analogy.

where τ_x is the component of frictional stress in the x direction, p_y is the y component of the pressure exerted by the matrix on the sphere, and α is the coefficient of friction.

It is shown in Figure 3 that the value of p_y is constant and equal to the radial pressure p exerted by the matrix on the bead due to the shrinkage of the resin on curing and to the difference in the coefficients of thermal expansion of the matrix and the filler.

The pressure at the matrix–filler interface is identical to a hydrostatic pressure and therefore does not depend on the direction in which it is measured.

The force transferred from the matrix to the bead over the element of length dx is

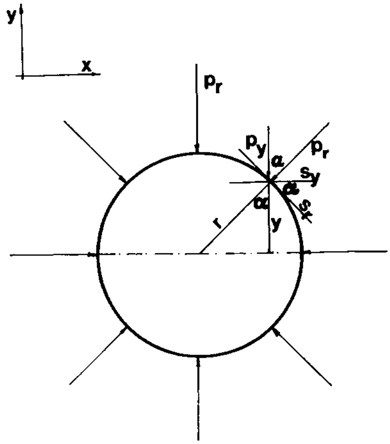
$$dF = 2\pi y \tau_x dx. \tag{2}$$

By combining eqs. (1) and (2), a differential equation is obtained which gives the load distribution in the bead:

$$\frac{dF}{dx} = 2\pi y p \alpha. \tag{3}$$

In order to solve eq. (3), y was expressed in terms of the radius of the sphere r and x, that is,

$$y = \sqrt{2rx - x^2}. \tag{4}$$



$$p_r s_r \cos\alpha = p_y s_y$$

$$\frac{s_y}{s_r} = \cos\alpha$$

$$p_y = p_r$$

Fig. 3. Relationship between radial pressure exerted by the matrix on the filler particle and the y component of this pressure.

The dimensionless parameter x/r can be introduced in place of x whereby

$$\frac{dF}{d\left(\dfrac{x}{r}\right)} = 2\pi p\alpha r^2 \sqrt{2\left(\frac{x}{r}\right) - \left(\frac{x}{r}\right)^2}. \tag{5}$$

The integrated solution of this differential equation is given by eq. (6):

$$F = \pi p\alpha r^2 \left[\left(\frac{x}{r} - 1\right)\sqrt{2\left(\frac{x}{r}\right) - \left(\frac{x}{r}\right)^2} + \sin^{-1}\left(\frac{x}{r} - 1\right) + \frac{\pi}{2}\right]. \tag{6}$$

The stress distribution in the bead can now be obtained by dividing the value of the load in any particular position in the bead by the corresponding cross-sectional area. Thus,

$$\sigma = \frac{F}{\pi y^2} = p\alpha \frac{\left(\dfrac{x}{r} - 1\right)\sqrt{2\left(\dfrac{x}{r}\right) - \left(\dfrac{x}{r}\right)^2} + \sin^{-1}\left(\dfrac{x}{r} - 1\right) + \dfrac{\pi}{2}}{2\left(\dfrac{x}{r}\right) - \left(\dfrac{x}{r}\right)^2}. \tag{7}$$

Figure 4 shows the graphic representation of eq. (7) (lower curve) for $p\alpha =$ 1540 psi.

The average stress in the bead is equal to the area under the σ-versus-(x/r) curve:

$$\sigma_{\text{avg}} = \int_0^1 \sigma d\left(\frac{x}{r}\right) = 0.83 p\alpha. \tag{8}$$

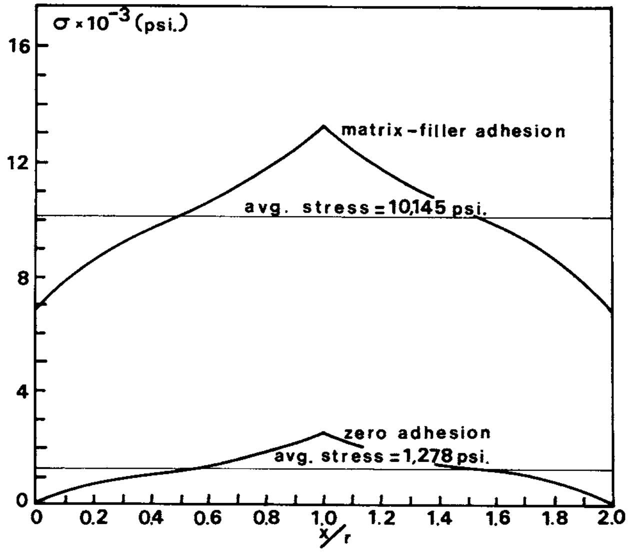


Fig. 4. Stress distribution in the particle of a spherical filler at the breaking point of a composite for the case of adhesion (upper curve) and no adhesion (lower curve).

The average stress in the bead at the breaking point of the composite is a function of a frictional stress $p\alpha$ only and is independent of the diameter of the sphere.

If it is assumed that the cross-sectional area of the composite filled with randomly oriented spheres is equal to one, then the portion of the cross-sectional area attributed to the beads is also V_b, where V_b is the volume fraction of the beads. The maximum load carried by the beads is then equal to

$$\sigma_b = \sigma_{\text{avg}} V_b = 0.83 p\alpha V_b. \tag{9}$$

The load carried by the matrix at the breaking point of the composite is

$$\sigma_m = K\sigma_{um}(1 - V_b) \tag{10}$$

where σ_{um} is the ultimate strength of the matrix and K is the relative change of the strength of the matrix due to the presence of the filler. Hajo and Toyashima[6] showed experimentally that at constant filler volume fraction, the strength of the composite was proportional to the reciprocal of the square root of the filler diameter. Since the maximum load carried by the filler is independent of the filler diameter, it follows that the ultimate strength of the matrix is dependent on $d^{-1/2}$. Hajo and Toyashima[6] explain their results on the basis of Griffith's theory and also refer to the work of Petch.[10]

In this study, Hajo and Toyashima's relationship was modified in order to describe the relative change of matrix strength in the presence of filler particles:

$$K = a + bd^{-1/2}$$

where a and b are constants.

Since in real situations the filler particles rarely have uniform diameters, there is a problem in the choice of diameter to be used in eq. (11). It was assumed that the strength of the composite was principally controlled by the largest diameters of the filler particles (which give the lowest strength of the composite).

Equations (9) and (10) may be combined to give an expression for the ultimate strength of the composite:

$$\sigma_{uc} = \sigma_b + \sigma_m = 0.83p\alpha V_b + K\sigma_{um}(1 - V_b). \tag{12}$$

According to eq. (12), the ultimate strength of the composite in case of no adhesion between the matrix and the filler is a linear function of V_b.

For constant V_b, the ultimate strength is a linear function of $d^{-1/2}$, which is in agreement with work of Hajo and Toyoshima.[6] Values of σ_{uc} extrapolated to $V_b = 0$ are equal to $K\sigma_{um}$ and depend on the diameters of the spheres. The value of σ_{uc} extrapolated to $V_b = 1$ is equal to 0.83 $p\alpha$ and is the same for all filler diameters. $V_b = 1$ represents an imaginary composite consisting of 100% glass spheres bonded together wherein the strength depends on the properties of the interface ($p\alpha$).

Effect of Interfacial Adhesion

When there is good adhesion between the spheres and the matrix, stress is transferred from the matrix to the filler in two ways: through shear stresses at the matrix–filler interface and through the x component of the tensile stress at the matrix–filler interface. The maximum stress in the bead is reached when the shear stress in the x direction, τ_x, reaches the ultimate shear strength of the matrix and the tensile stress at the interface attains the filler–matrix bond strength.

As indicated by eq. (9), the load carried by the beads in case of no adhesion is proportional to V_b, viz.,

$$\sigma_b = c_1 V_b \tag{13}$$

where c_1 is a constant. Similar dependence is expected in the case of matrix–filler adhesion:

$$\sigma_b = c_2 V_b \tag{14}$$

where c_2 is a constant.

When the matrix–filler bond fails, the additional load placed on the matrix is, therefore, given by

$$\Delta\sigma_b = (c_2 - c_1)V_b \tag{15}$$

so that (a) at low volume fractions of the beads, the additional load placed on the matrix is small and does not cause catastrophic failure; the strength of the composite is the same as in the case of no adhesion; and (b) at high volume fraction, the additional load placed on the matrix is large and leads to catastrophic failure of the composite.

Since the case of low volume fraction V_b is similar to the previous case with no adhesion, only the case of high volume fraction V_b need be discussed.

The stress in the bead is the sum of a stress transferred by shearing and stress transferred by the x component of the tensile stress at the matrix–filler interface. It is assumed that the maximum stress transferred by the x component of the tensile stress is the same in any particular position in the bead and equal to the strength of the matrix–bead adhesion σ_a.

The stress transferred from the matrix to the sphere by shearing can be calculated in the manner described in part 1. Equations (2) to (8) apply here, where the shear strength of the matrix τ_m is substituted for the frictional stresses $p\alpha$.

Modified eq. (7) has the form

$$\sigma_s = \tau_m \frac{\left(\frac{x}{r} - 1\right)\sqrt{2\left(\frac{x}{r}\right) - \left(\frac{x}{r}\right)^2} + \sin^{-1}\left(\frac{x}{r} - 1\right) + \frac{\pi}{2}}{2\left(\frac{x}{r}\right) - \left(\frac{x}{r}\right)^2} \tag{16}$$

where σ_s is the stress transferred by shearing.

The total stress in the bead is a sum of a stress transferred by shearing and by the x component of the tensile stress at a matrix–filler interface:

$$\sigma = \sigma_a + \sigma_s = \sigma_a + \tau_m \frac{\left(\frac{x}{r} - 1\right)\sqrt{2\left(\frac{x}{r}\right) - \left(\frac{x}{r}\right)^2} + \sin^{-1}\left(\frac{x}{r} - 1\right) + \frac{\pi}{2}}{2\left(\frac{x}{r}\right) - \left(\frac{x}{r}\right)^2}. \tag{17}$$

Figure 4 (upper curve) shows the stress distribution in the bead when $\tau_m = 4150$ psi and $\sigma_a = 6700$ psi where the value of σ_a may be estimated from the experimental data.

The average stress in the bead is equal to the area under the σ-versus-x/r curve, viz.,

$$\sigma_{\text{avg}} = \int_0^1 \sigma d\left(\frac{x}{r}\right) = 0.83\tau_m + \sigma_a. \tag{18}$$

The load carried by the beads at the point of failure of the composite is then given by

$$\sigma_b = (\sigma_a + 0.83\tau_m)V_b. \tag{19}$$

It was assumed that the maximum tensile stress in the x direction at the matrix–filler interface was equal to the interfacial adhesion σ_a. Due to stress concentration, the average tensile stress in the matrix is usually lower than the stress at the filler–matrix interface. The stress concentration factor was defined as

$$S = \frac{\text{average tensile stress in matrix at breaking point of the composite}}{\text{tensile stress in } x \text{ direction at matrix–filler interface } (=\sigma_a)} \tag{20}$$

The load supported by the matrix is given by

$$\sigma_m = \sigma_a S(1 - V_b). \tag{21}$$

Equations (19) and (21) combined give an expression for the ultimate strength of composites filled with spheres (for the case of high V_b and matrix–filler adhesion):

$$\sigma_{uc} = (\sigma_a + 0.83\tau_m)V_b + \sigma_a S(1 - V_b). \tag{22}$$

For low volume fraction V_b, eq. (12) applies, viz.,

$$\sigma_{uc} = 0.83p\alpha V_b + K\sigma_{um}(1 - V_b). \tag{12}$$

There is one particular value of V_b at which σ_{uc} calculated from eq. (12) is equal to σ_{uc} calculated from eq. (21). At this particular volume fraction of the filler, the composite exhibits minimum ultimate strength. By equating the right sides of eqs. (12) and (22), the value of the volume fraction of the filler at minimum composite strength can be calculated:

$$V_{b\,\mathrm{min}} = \frac{K\sigma_{um} - \sigma_a S}{\sigma_a + 0.83\tau_m + K\sigma_{um} - \sigma_a S - 0.83p\alpha}. \tag{23}$$

Thus, eq. (12) applies for $0 < V_b < V_{b\,\mathrm{min}}$ and eq. (22) for $V_{b\,\mathrm{min}} < V_b < 1$.

Values of σ_{uc} extrapolated to $V_b = 1$ depend only on τ_m and are independent of the filler diameters. On the other hand, values of σ_{uc} extrapolated to $V_b = 0$ are dependent on the diametrs of the particles of the spherical filler.

RESULTS AND DISCUSSION

As was shown in the theoretical treatment, the ultimate strength of the composite filled with spherical filler is a linear function of V_b. Coefficients of these straight lines were obtained from the experimental results using the least-squares fitting. The results for tensile samples are shown in Table II. Since it is postulated that the value of σ_{uc} extrapolated to $V_b = 1$ is independent of the diameter of the filler particles, the average value of σ_{uc} at

TABLE II
Ultimate Tensile Strength of the Composites Extrapolated to $V_b = 0$ and $V_b = 1$ (First Fitting)

Stock size of beads	Diameter, mm	Ultimate strength σ_{uc} at $V_b = 0$, psi	Ultimate strength σ_{uc} at $V_b = 1$,[a] psi
1721	0.354–0.177	5870	827
1922	0.250–0.149	7060	480
2024	0.210–0.105	7460	501
2429	0.105–0.053	7148	260
2900	–0.053	8515	440
3000	–0.044	8180	720

[a] The average value of σ_{uc} at $V_b = 1$ is 540 psi.

TABLE III
Ultimate Tensile Strength of the Composites
Extrapolated to $V_b = 0$ and $V_b = 1$ (Second Fitting)[a]

Stock size of beads	Diameter, mm	Ultimate strength σ_{uc} at $V_b = 0$, psi	Ultimate strength σ_{uc} at $V_b = 1$, psi
1721	0.354–0.177	5950	
1922	0.250–0.149	7044	
2024	0.210–0.105	7450	540 psi for all sizes
2429	0.105–0.053	7071	
2900	–0.053	8049	
3000	–0.044	8227	

[a] The standard deviation of the fitting is ±260 psi.

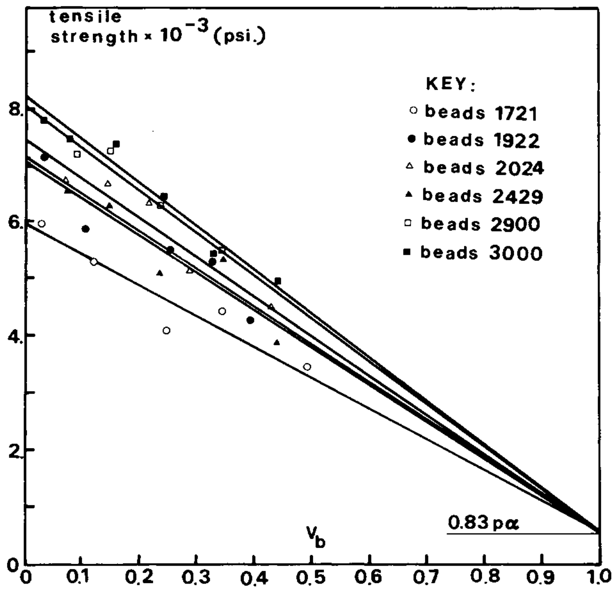


Fig. 5. Tensile strength of a composite as a function of the volume fraction of the filler for different sphere diameters (refer to Table I).

$V_b = 1$ was calculated to be 540 psi, and the least-squares fitting was repeated again with the assumption that the point ($\sigma_{uc} = 540$ psi, $V_b = 1$) is common for all lines. Results of this fitting are shown in Table III and graphically in Figure 5. The value of σ_{uc} at $V_b = 1$ was also calculated from eq. (12):

$$\sigma_{uc} \text{ (at } V_b = 1) = 0.83p\alpha = 0.83 \times 1540 \text{ psi}$$
$$= 1278 \text{ psi}$$

which is greater than the 540 psi average value obtained from Table II.

The values of K were determined by dividing the values of σ_{uc} at $V_b = 0$ by the ultimate strength of the matrix. The values of K versus $d^{-1/2}$ are

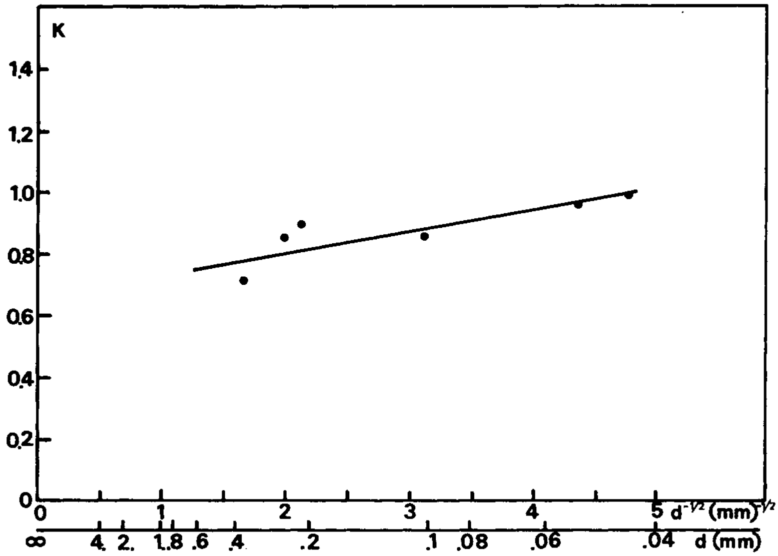


Fig. 6. Relative change of ultimate tensile strength of the matrix due to the presence of the spheres as a function of the diameters of the largest beads present in the sample.

plotted in Figure 6. It is apparent that small particles weaken the matrix to a lesser extent than large particles.

Figure 7 shows the relationship between the ultimate strength of a composite filled with glass spheres and the volume fraction when there is adhesion between the matrix and the filler. Lines representing the ultimate strength of the composite in the low volume fraction region were taken directly from Figure 5 (case of no adhesion). The assumption that the ultimate strength of the composite at low volume fraction V_b is the same for the case of matrix–filler adhesion and no adhesion is in good agreement with the experimental results. Test samples were prepared in the region $V_b = 10\%$ to $V_b = 50\%$. Outside this region samples could not be prepared so that the theoretical relationship could not be compared over the full range of V_b. However, the assumptions that σ_{uc} at $V_b = 1$ does not depend on the diameter of the spheres and that σ_{uc} is a linear function of V_b is in agreement with the experimental results.

The value of σ_{uc} at $V_b = 1$ for the case of the matrix-filler adhesion is approximately 10,100 psi. Assuming $\tau_m = 4150$ psi, σ_a was calculated to be 6700 psi from eq. (22). By examining the electron-scanning photomicrographs of the fracture surface of the composite samples in Figure 8, it is expected that σ_a is only slightly smaller than σ_m since there are spots on the surface of the beads covered by the resin (for such spots $\sigma_a > \sigma_{um}$).

Figure 9 shows the dependence of the ultimate strength of the composite filled with glass microspheres (stock size 2429) on the volume fraction of the beads for different values of the matrix–filler adhesion bond strength as determined from eq. (22) (assuming $S = 0.57$ as calculated from Figure 7 and eq. (22) using $\tau_m = 4150$ psi).

Similar results were obtained for the ultimate flexural strength of the composite. Tables IV and V show the results of first and second curve fitting. Figure 10 shows the dependence of the flexural strength on volume fraction V_b for the case of no adhesion, and Figure 11 shows the same de-

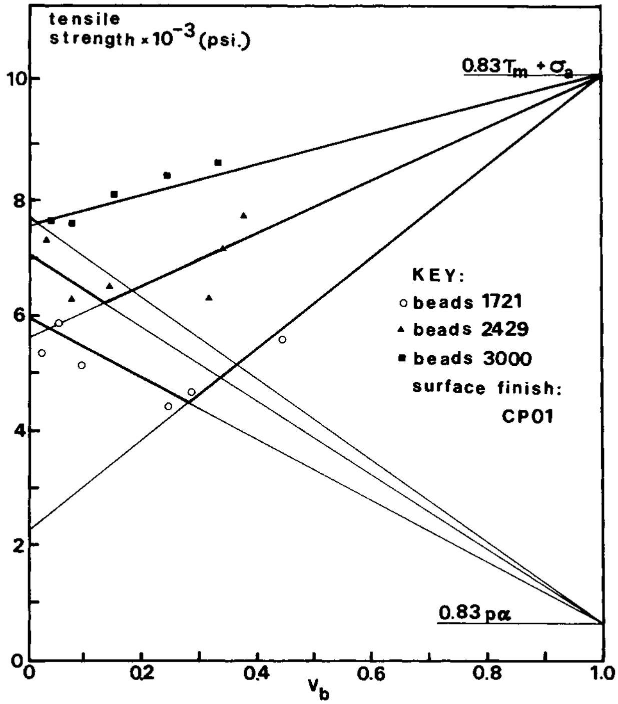
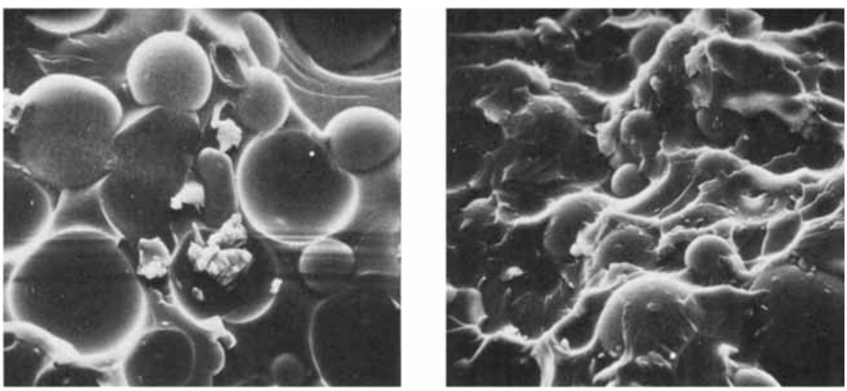


Fig. 7. Tensile strength of composite as a function of the volume fraction of the filler for different stock sizes of the filler when there is adhesion between the matrix and the filler.



(a) (b)

Fig. 8. Electron-scanning photomicrograph of the fracture surface of the polyester–glass microbead (stock size 3000) composite; (a) untreated beads; (b) treated beads. Magnification 850×.

TABLE IV
Ultimate Flexural Strength of the Composites
Extrapolated to $V_b = 0$ and $V_l = 1$ (First Fitting)

Stock size of beads	Diameter, mm	Ultimate strength σ_{uc} at $V_b = 0$, psi	Ultimate strength σ_{uc} at $V_b = 1$,[a] psi
1721	0.354–0.177	13,590	3430
1922	0.250–0.149	16,690	4510
2024	0.210–0.105	16,410	5640
2429	0.105–0.053	20,450	233
2900	–0.053	21,940	1010
3000	–0.044	21,850	1100

[a] The average value of σ_{uc} at $V_b = 1$ is 2650 psi.

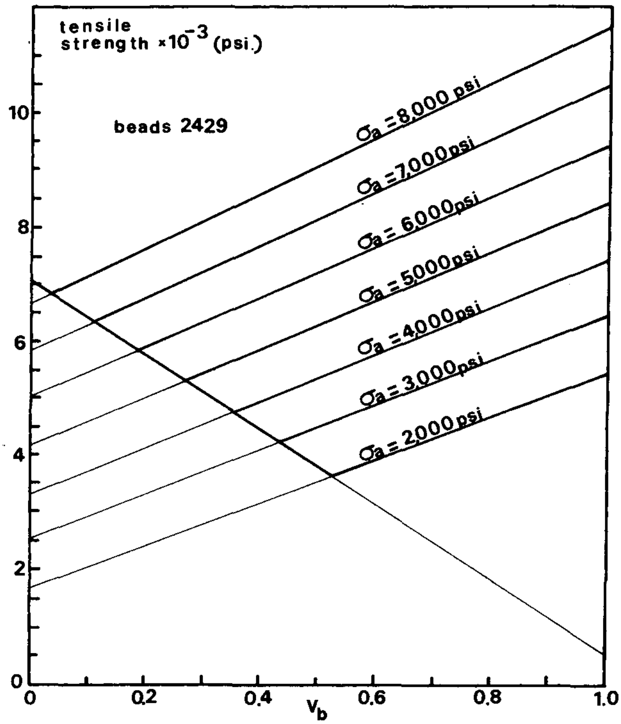


Fig. 9. Tensile strength of polyester resin–glass bead composite as a function of the volume fraction of the beads for different values of matrix–filler bond strength, as calculated from eq. (22).

pendence when there is adhesion. Values of K (Fig. 12) are slightly higher than those obtained from the tensile experiments.

CONCLUSIONS

It has been shown that the ultimate strength of a composite filled with spherical filler particles is a linear function of V_b and, for constant volume

TABLE V
Ultimate Flexural Strength of the Composites
Extrapolated to $V_b = 0$ and $V_b = 1$ (Second Fitting)[a]

Stock size of beads	Diameter, mm	Ultimate strength σ_{uc} at $V_b = 0$, psi	Ultimate strength σ_{uc} at $V_b = 1$, psi
1721	0.354–0.177	13,780	
1922	0.250–0.149	17,090	
2024	0.210–0.105	16,880	2650 psi for all sizes
2429	0.105–0.053	19,900	
2900	–0.053	21,560	
3000	–0.044	21,410	

[a] Standard deviation of the fitting is ±520 psi.

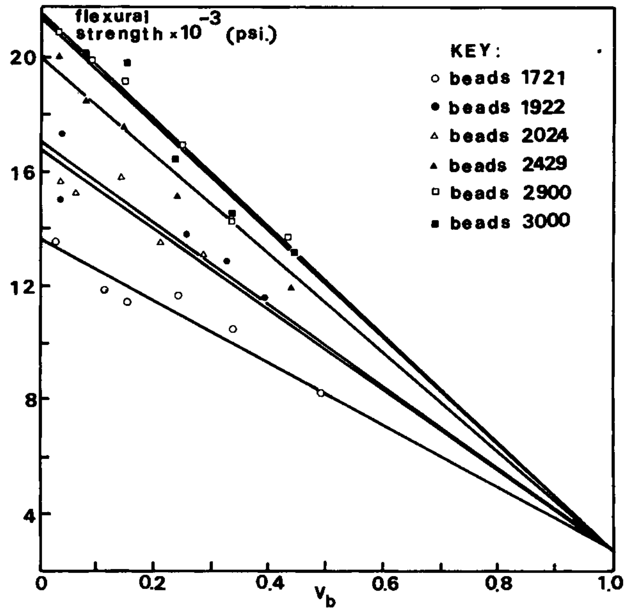


Fig. 10. Flexural strength of composite as a function of the volume fraction of the filler for different stock sizes of the beads (refer to Table I) (no matrix–filler adhesion).

fraction of the filler, is inversely proportional to the square root of the sphere diameter, i.e., $d^{-1/2}$.

The ultimate strength of the matrix alone is altered by the presence of the beads and can be obtained from the plot σ_{uc} versus V_b by extrapolating the values of σ_{uc} to $V_b = 0$. The slope of the K-versus-$d^{-1/2}$ plot as well as the values of K probably depend on the notch sensitivity of the matrix.

In the case of filler–matrix adhesion, the ultimate strength of the composite as a function of V_b is given by two straight lines: (a) for small volume fraction V_b—identical to that for the case of no adhesion, negative slope; the ultimate strength of the composite decreases with increase in the volume fraction of the filler; (b) for large volume fraction V_b—positive slope; strength of the composite increases with increase of the volume fraction of the filler.

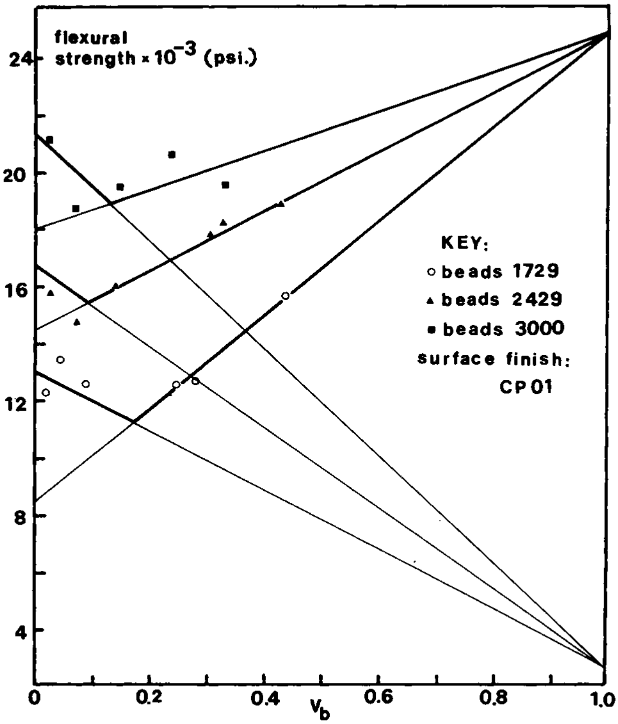


Fig. 11. Flexural strength of composite as a function of the volume fraction of the beads for different stock sizes of the beads (matrix–filler adhesion).

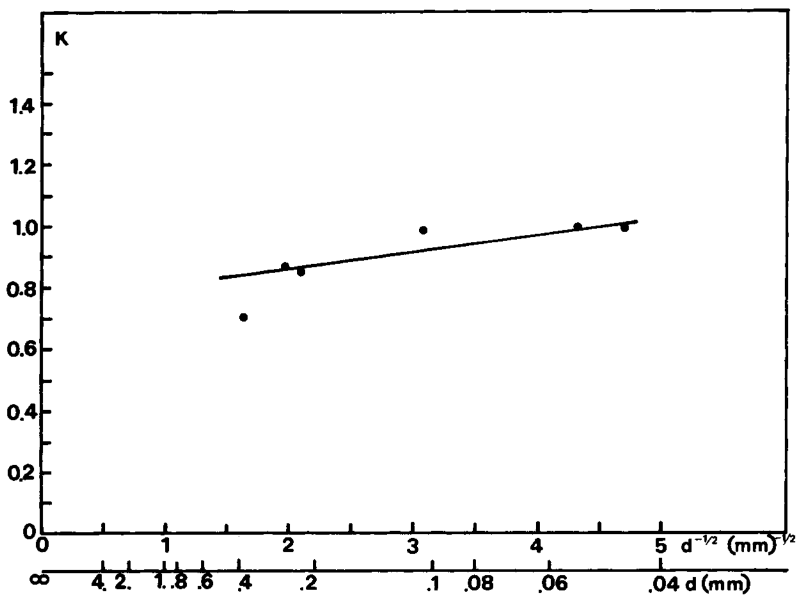


Fig. 12. Relative change of flexural strength of the resin due to the presence of the filler as a function of the diameters of the largest beads present in the sample.

Similar relationships were obtained for both ultimate tensile and ultimate flexural strengths. Thus, the strength dependence follows the same pattern as observed for fiber-filled composites.

Although the results obtained in this treatment are valid only for composites filled with spherical fillers, the derived relationships with minor modifications may also be employed to characterize composites containing fillers with irregular shapes.

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Nomenclature

d	diameter of the spherical particle
F	load carried by the bead
K	relative change of the strength of the matrix due to the presence of the filler
P	radial pressure exerted by the matrix on the filler particles
p_y	y component of the pressure exerted by the matrix on the filler particles
r	the radius of the sphere
S	stress concentration factor
V_b	volume fraction of the beads
$V_{b\,\mathrm{min}}$	volume fraction of the beads at which the composite strength is at its lowest value
α	coefficient of friction
τ_x	x component of frictional stresses at the matrix–filler interface
σ	stress in the bead
σ_a	matrix–filler adhesion bond strength
σ_{avg}	average stress in the bead
σ_b	maximum load carried by the filler
σ_m	maximum load carried by the matrix
σ_{uc}	ultimate strength of the composite
σ_{um}	ultimate strength of the matrix
σ_s	stress transferred from the matrix to the spfere by shearing at the interface

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